Quantum physics. Department of physics. 7th semester.

Lesson №12. ***Perturbation theory (PT): time-independent PT for nondegenerate levels, time-independent degenerate PT.***

1. Perturbation theory.

$$\hat{H} = \hat{H}_0 + \hat{V}; \quad \hat{H}_0 \psi_n^{(0)} = E_n^{(0)} \psi_n^{(0)};$$

$\hat{H}_0$ – basic Hamiltonian, which exact solution of time-independent Schrodinger's is known, $\hat{V}$ – perturbation operator.

$\dfrac{\partial \hat{V}}{\partial t} = 0$ – time-independent PT, $\dfrac{\partial \hat{V}}{\partial t} \neq 0$ – time-dependent PT (see lesson № 13).

2. Time-independent PT for nondegenerate levels.

$$\hat{H}\psi = E\psi,$$

$$E = E_n^{(0)} + E_n^{(1)} + E_n^{(2)} + \ldots; \quad \psi = \psi_n^{(0)} + \psi_n^{(1)} + \psi_n^{(2)} + \ldots.$$

$$\begin{cases} E_n^{(1)} = V_{nn}; \\ \psi_n^{(1)} = \sum\limits_{m \neq n} \dfrac{V_{mn}}{E_n^{(0)} - E_m^{(0)}} \psi_m^{(0)}; \end{cases} \qquad E_n^{(2)} = \sum_{m \neq n} \frac{|V_{mn}|^2}{E_n^{(0)} - E_m^{(0)}};$$

$V_{mn} = \left(\psi_m^{(0)}, \hat{V}\psi_n^{(0)}\right)$ – matrix elements of perturbation operator.

$|V_{mn}| << |E_m^{(0)} - E_n^{(0)}|$ –PT's usability condition.

Task 1. For a particle in an infinitely deep potential well of width $a \; (0 < x < a)$ find in the first order of PT displacement of energy levels under the influence of perturbations of the form $V(x) = \dfrac{V_0}{a}\left(a - |2x - a|\right)$. (ГКК № 8.1 (a))

Task 2. Operator's matrix elements of perturbation, which influence on the linear oscillator with fundamental (natural) frequency ω, have a form $V_{mn} = \alpha\delta_{m,n+1} + \beta\delta_{m,n-1}$. Find corrections $E_n^{(1)}$, $E_n^{(2)}$ to oscillator's energy, in first two orders of perturbation theory.

3. Time-independent degenerate PT. Secular equation.

$$\hat{H}_0\psi_n^{(0)} = E_n^{(0)}\psi_n^{(0)}; \quad \underbrace{\psi_n^{(0)}, \psi_{n'}^{(0)}, \ldots, \psi_{n''}^{(0)}}_{s} \text{ } (s\text{– fold degenerate level})$$

$$\sum_{n'=1}^{s}\left(V_{nn'} - E^{(1)}\delta_{n,n'}\right)C_{n'}^{(0)} = 0.$$

Secular equation

$$Det\left(V_{nn'} - E^{(1)}\delta_{n,n'}\right) = 0,$$

$\psi = \sum_{n=1}^{s} C_n^{(0)}\psi_n^{(0)}$ – "good" linear combinations of unperturbed states

Task 3. Define first order corrections for energy and to the first approximation to eigenvalue "good" linear combinations of unperturbed states for doubly degenerated level $s = 2$.

Task 4. Flat rotator with inertia momentum I and dipole momentum $\vec{d}$ is placed in the uniform electric field, lying in the plane of rotation. Find in the first two orders of perturbation theory shift and splitting of energy levels of exited states with $m \neq 0$ of the rotator.

For the solving this problem we need to use a revised formula of PT for degenerated levels, which takes into account in second order of PT matrix elements for transitions to states with another energies (see LL§39, formula (39.4))

$$\sum_{n'=1}^{s}\left(V_{nn'} + \sum_k \frac{V_{nk}V_{kn'}}{E_n^{(0)} - E_k^{(0)}} - E^{(2)}\delta_{n,n'}\right)C_{n'}^{(0)} = 0.$$

Hometask. НКК 8.1 (б), 8.3, 8.4, 8.5, 8.9, ЛЛ 39(1) (find WF), 8.10 (complete the solution of the problem), 8.11.

LL – Landau LD, Lifshits EM, Quantum Mechanics

НКК- Halitskii E.M., Karnakov B.M., Kohan V.I. Problems in Quantum Physics, 1981

ГКК - Галицкий Е.М., Карнаков Б.М., Коган В.И. Задачи по квантовой механике, 1981; ЛЛ – Ландау Л.Д., Лифшиц Е.М. Квантовая механика